AXIAL LOAD

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Axial Load 121

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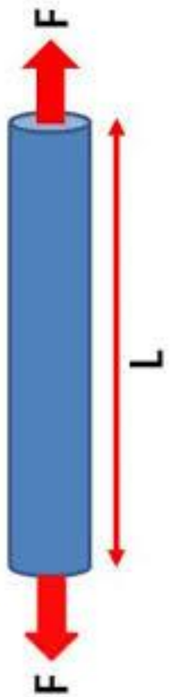

4.6 Thermal Stress

- A change in temperature can cause a body to change its dimensions.
- if the temperature increases, the body will expand, whereas if the temperature decreases, it will contract.
- This expansion or contraction is *linearly* related to the temperature increase or decrease that occurs.
- If the material is homogeneous and isotropic, it has been found from experiment that the displacement of a member having a length *L* can be calculated using the formula

$$\boxed{\delta_T = \alpha \Delta T L}$$

α = liner coefficient of thermal expansion. Unit measure strain per degree of temperature: $1/^{o}C$ (Celsius) or $1/^{o}K$ (Kelvin)

ΔT = algebraic change in temperature of member

$\boldsymbol{\delta_t}$ = algebraic change in length of member.

L = the original length of the member

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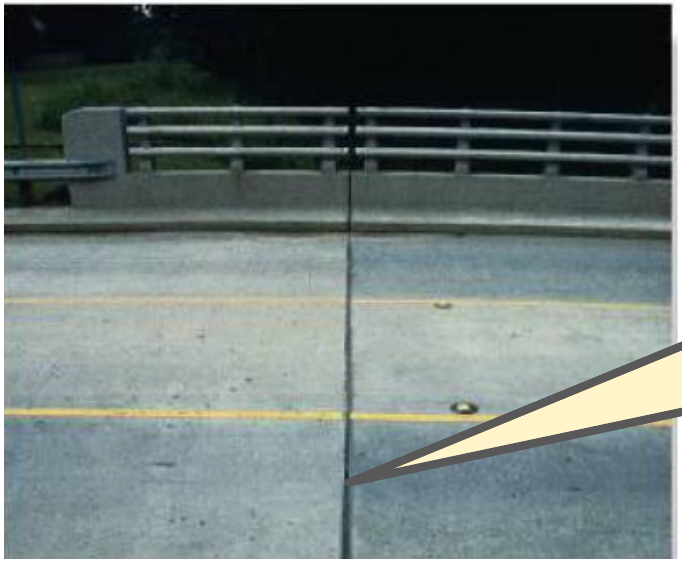
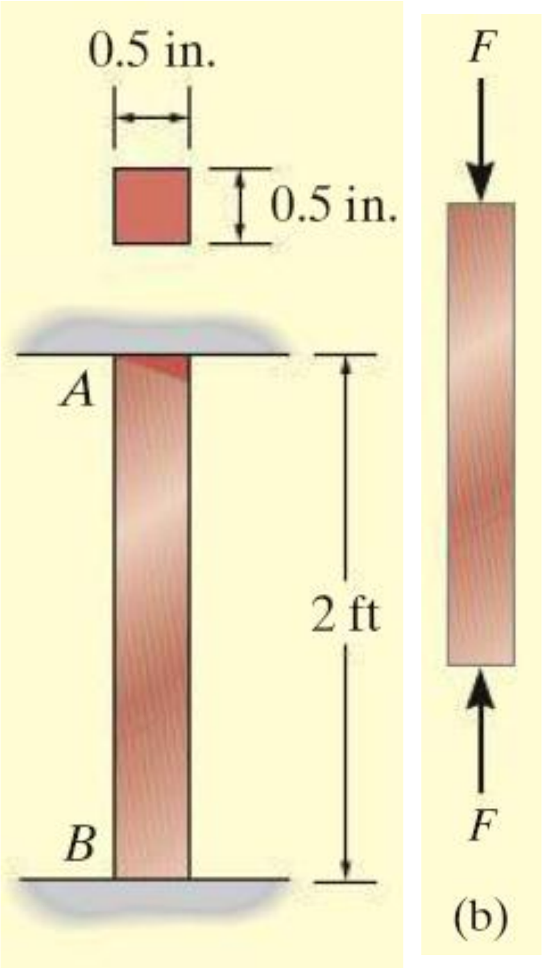


Most traffic bridges are designed with expansion joints to accommodate the thermal movement of the deck and thus avoid any thermal stress.

Long extensions of ducts and pipes that carry fluids are subjected to variations in climate that will cause them to expand and contract. Expansion joints, such as the one shown, are used to mitigate thermal stress in the material.

The A-36 steel bar shown in Fig. 4–17*a* is constrained to just fit between two fixed supports when $T_1 = 60°F$. If the temperature is raised to $T_2 = 120°F$, determine the average normal thermal stress developed in the bar.



SOLUTION

- Equilibrium

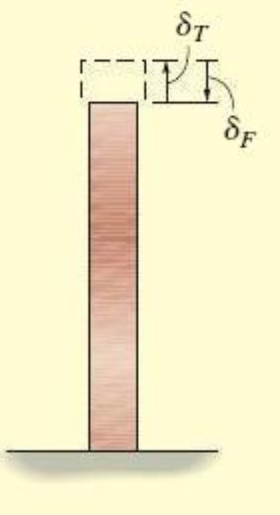
.

$$+\uparrow \Sigma F_y = 0;$$

$$F_A = F_B = F$$

The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at A that occurs, Fig. 4–17*c*, is counteracted by the force **F** that is required to push the bar δ_F back to its original position. The compatibility condition at A becomes



(c)

$$(+\uparrow) \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

Load-Displacement. Applying the thermal and load–displacement relationships, we have

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

Thus, from the data on the inside back cover,

$$\begin{aligned} F &= \alpha \Delta T A E \\ &= [6.60(10^{-6})/°\text{F}](120°\text{F} - 60°\text{F})(0.5\ \text{in.})^2\,[29(10^3)\ \text{kip/in}^2] \\ &= 2.871\ \text{kip} \end{aligned}$$

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Since **F** also represents the internal axial force within the bar, the average normal compressive stress is thus

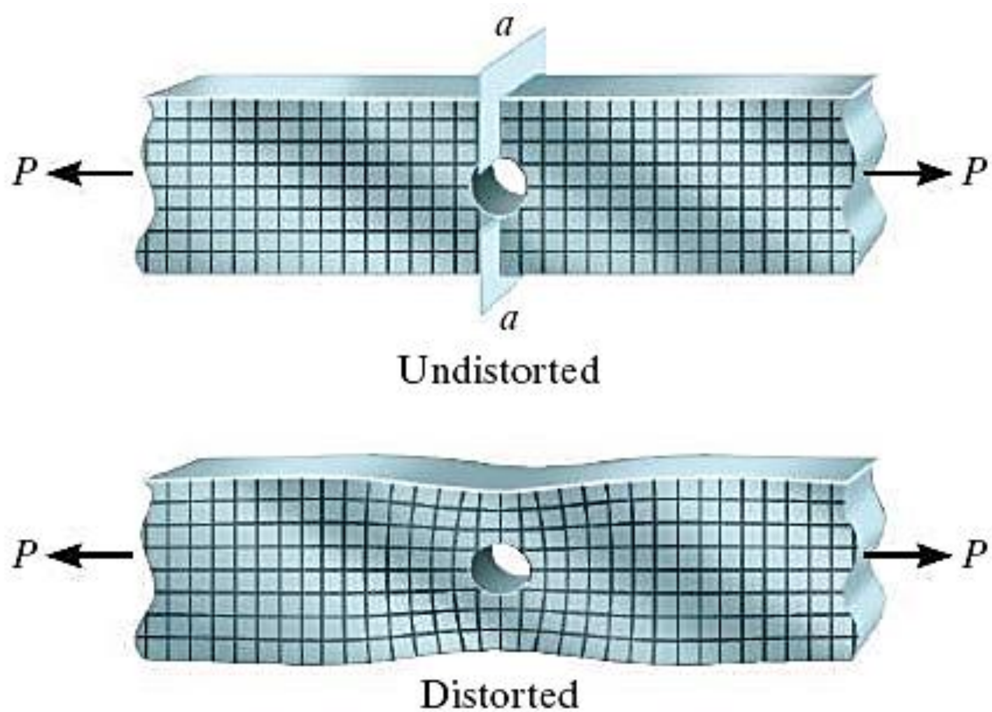
$$\sigma = \frac{F}{A} = \frac{2.871 \text{ kip}}{(0.5 \text{ in.})^2} = 11.5 \text{ ksi} \qquad \textit{Ans.}$$

NOTE: From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members.

Strength of Materials: Second Class

4.7 Stress Concentrations

- Consider the bar in Fig. 4–20 a, which is subjected to an axial force P.
- The maximum normal stress in the bar occurs on section $a-a$, which is taken through the bar's *smallest* cross-sectional area.



(a)

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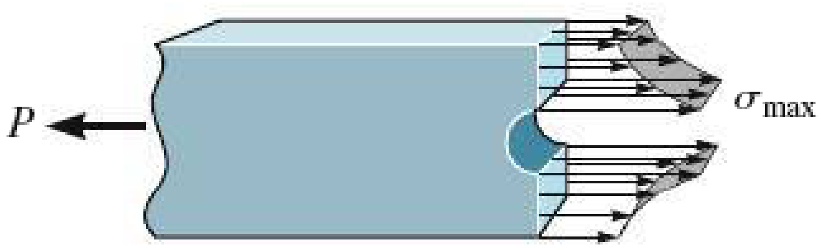
- the stress distribution acting on this section can be determined either from a mathematical analysis, using the theory of elasticity, or experimentally by measuring the strain normal to section $a - a$ and then calculating the stress using Hooke's law, $\sigma = E\epsilon$.

- Regardless of the method used, the general shape of the stress distribution will be like that shown in Fig. 4–20 b .



Actual stress distribution
(b)

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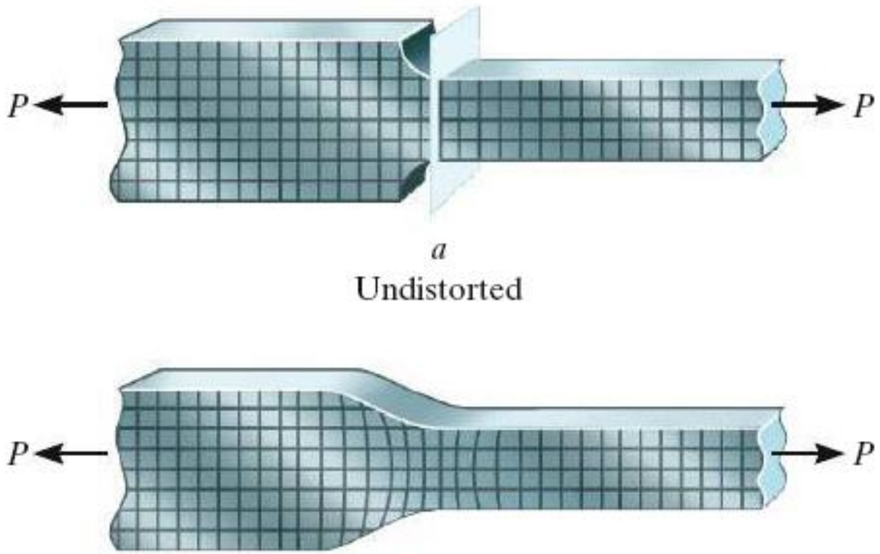
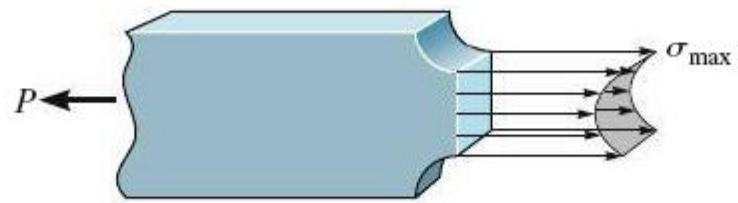
- In similar manner, if the bar has a reduction in its cross section, achieved using shoulder fillets as in Fig. 4–21 *a* ,
- then again the maximum normal stress in the bar will occur at the *smallest* cross-sectional area, section *a* – *a* , and the stress distribution will look like that shown in Fig. 4–21 *b* .



(a)



(b)

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- In both of these cases, *force equilibrium* requires the magnitude of the *resultant force* developed by the stress distribution to be equal to *P*.

$$P = \int_A \sigma \, dA$$

- This integral *graphically* represents the total *volume* under each of the stress-distribution diagrams shown in Fig. 4–20 *b* or Fig. 4–21 *b*.

- The resultant **P** must act through the *centroid* of each *volume*.

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In engineering practice, actual stress distribution not needed, only *maximum stress* at these sections must be known. Member is designed to resist this stress when axial load **P** is applied.

K is defined as a ratio of the maximum stress to the average stress acting at the smallest cross section:

$$\boxed{K = \frac{\sigma_{\max}}{\sigma_{\text{avg}}}} \qquad (4\text{–}6)$$

K is independent of the bar's geometry and the type of discontinuity, only on the bar's geometry and the type of discontinuity.

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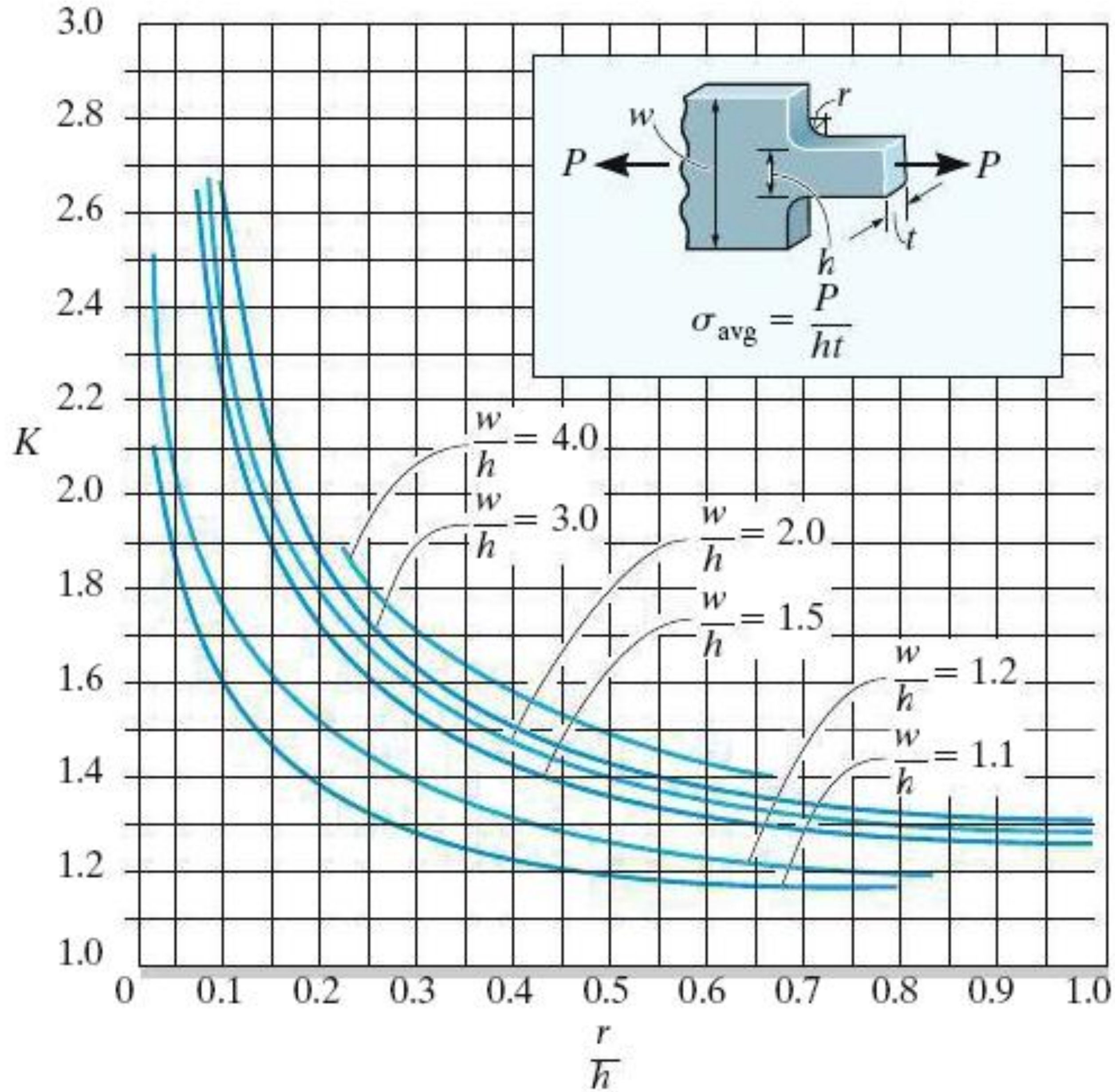
- As size *r* of the discontinuity is decreased, stress concentration is increased.

- It is important to use stress-concentration factors in design when using brittle materials, but not necessary for ductile materials

- Stress concentrations also cause failure structural members or mechanical elements subjected to *fatigue loadings*

AXIAL LOAD



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r/h
w/h = 4.0
w/h = 3.0
w/h = 2.0
w/h = 1.5
w/h = 1.2
w/h = 1.1
P
P
w
r
h
t
σavg = P/ht

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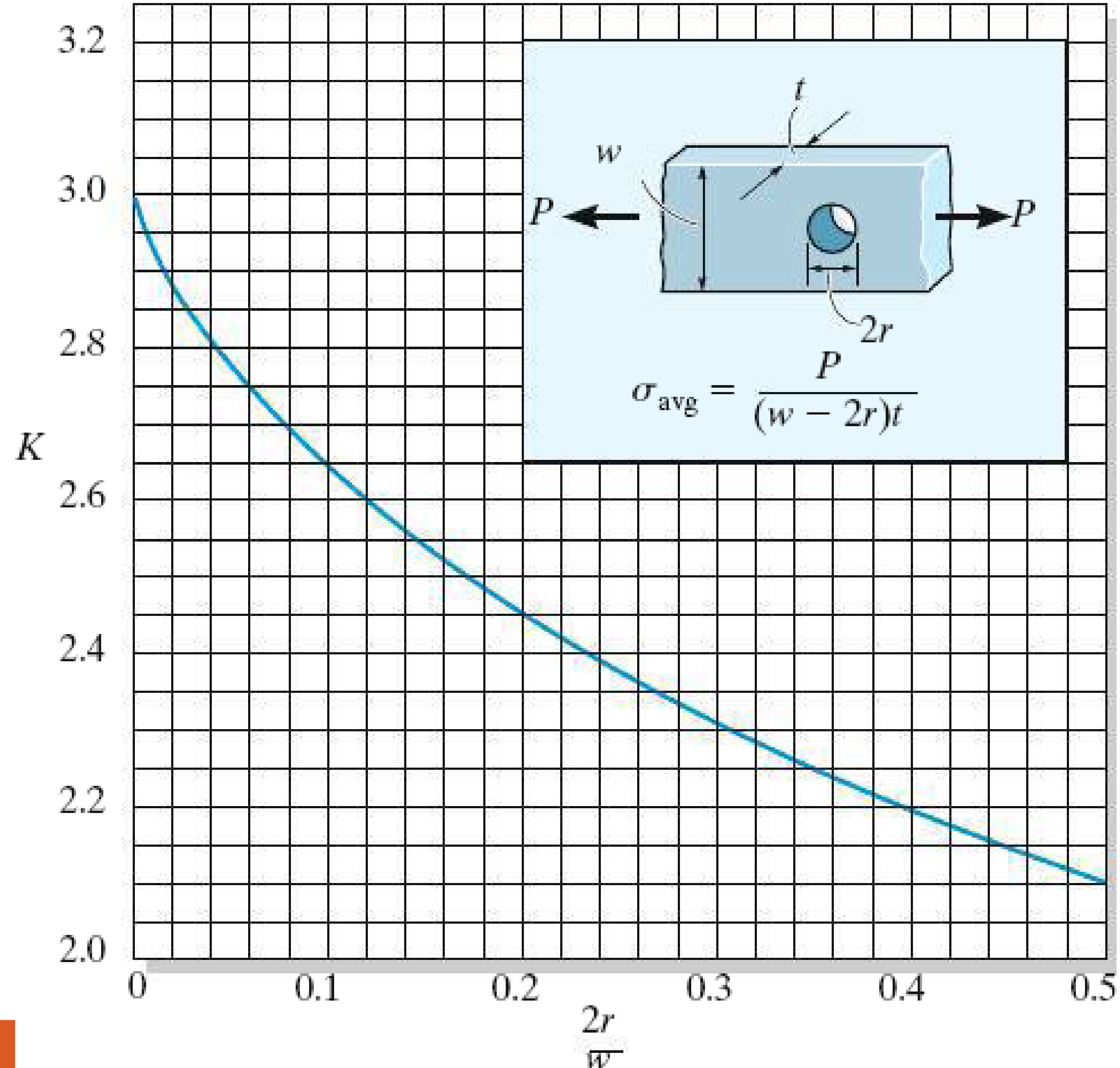
3.2
3.0
2.8
2.6
2.4
2.2
2.0
K
0
0.1
0.2
0.3
0.4
0.5
2r/w
t
w
P
P
2r
$\sigma_{avg} = \frac{P}{(w - 2r)t}$

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This saw blade has grooves cut into it in order to relieve both the dynamic stress that develops within it as it rotates and the thermal stress that develops as it heats up. Note the small circles at the end of each groove. These serve to reduce the stress concentrations that develop at the end of each groove.

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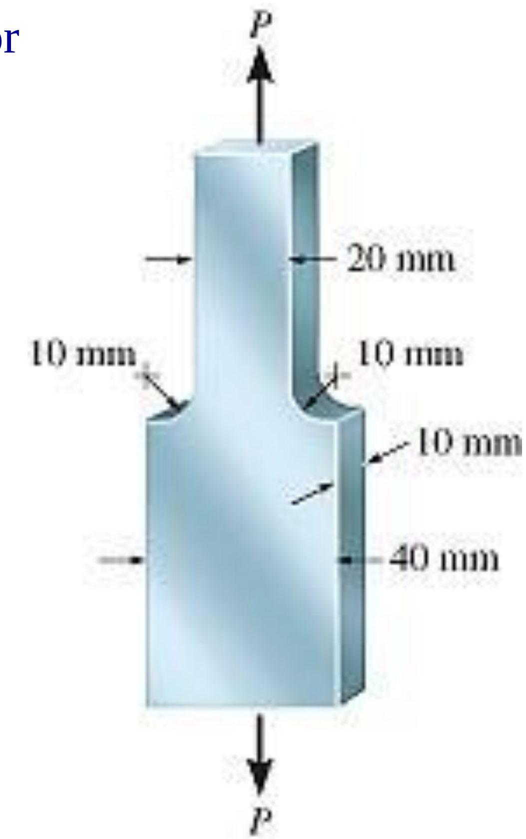
Ex1:- Steel bar shown below has allowable stress, σ_{allow} = 115 MPa. Determine largest axial force **P** that the bar can carry.

Solution:-

Because there is a shoulder fillet, stress-concentrating factor determined using the graph below



Calculating the necessary geometric parameters yields

$$\frac{r}{n} = \frac{10\text{ mm}}{20\text{ mm}} = 0.50\ , \quad \frac{w}{h} = \frac{40\text{ mm}}{20\text{ mm}} = 2$$

Thus, from the graph 4-23, $K = 1.4$

Average normal stress at *smallest* x-section,

$$\sigma_{avg} = \frac{P}{(20\text{ mm})(10\text{ mm})} = 0.005\,P\ \text{N/mm}^2$$

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Applying Eqe 4-7 with $\sigma_{allow} = \sigma_{max}$ yields

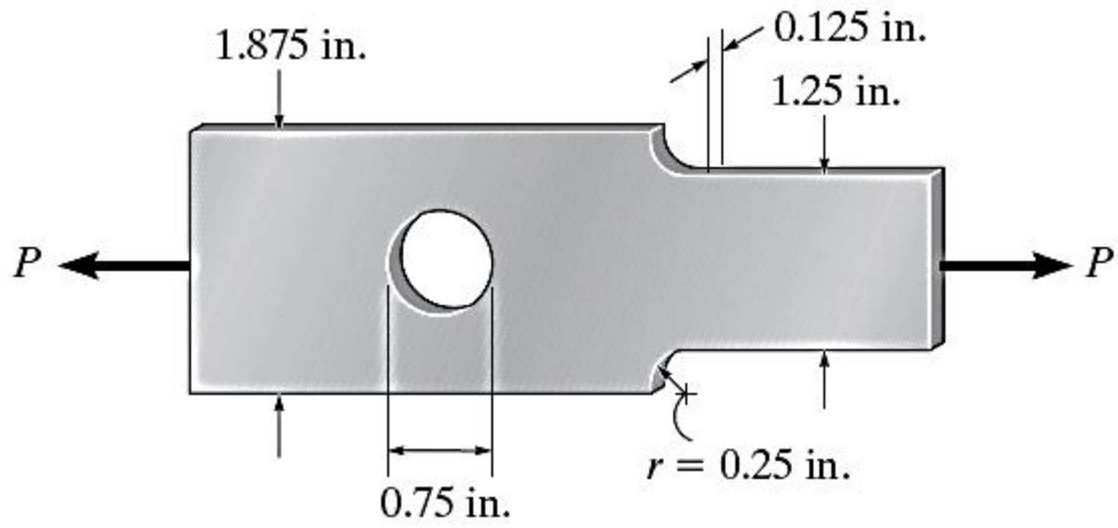
$$\sigma_{allow} = K\,\sigma_{max}$$

$$115 \text{ N/mm}^2 = 1.4(0.005P)$$

$$P = 16.43(10^3) \text{ N} = 16.43 \text{ kN}$$

Homework

1:- Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of σ_{allow}= 21 ksi.



Homework

2. Determine the maximum normal stress developed in the bar when it is subjected to a tension of P = 2 kip.